

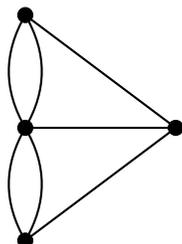
(Read the directions and follow the instructions)

The Bridges of Königsberg: Through the middle of the German city of Königsberg, a wide river runs from east to west. In the middle of the river are two islands: The island of Gauss to the west and the island of Fermat to the east. There are two bridges connecting the island of Gauss to the north bank, and two bridges connecting the island of Gauss to the south bank. Additionally, there is one bridge connecting the island of Fermat to the north bank, and one bridge connecting the island of Fermat to the south bank. Finally, there is one bridge that connects the island of Gauss to the island of Fermat.

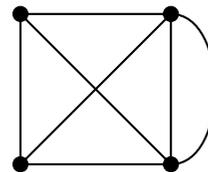
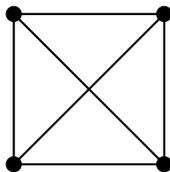
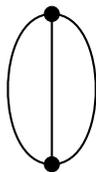
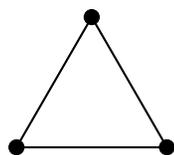
Exercise 1. In the space below, draw the city of Königsberg along with all its bridges. (There should be seven bridges in total).

Exercise 2. On a sunny day, the mathematician Leonhard Euler wants to take a walk around Königsberg in which he crosses each bridge exactly once. Suppose he starts from the south bank. Is such a walk possible? Try to draw one on the diagram you drew above.

You can simplify the problem by drawing a diagram with one point for every land mass and one line for every bridge:



The above image is called a **graph**. Below are some more examples of graphs:



Exercise 3. Draw four of your own graphs in the space below.

A path in a graph that goes over each line exactly once is called an **Euler Path**.

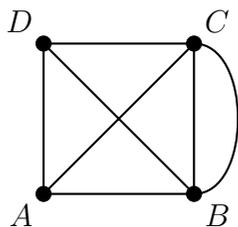
Exercise 4. Label each of the nine above graphs with a number. In the space below, say whether each graph has an Euler path or not.

| | | | | | | | | |
|--------------|--|--|--|--|--|--|--|--|
| Graph Number | | | | | | | | |
| Euler Path? | | | | | | | | |

Some graph vocabulary:

- A point is called a **vertex**. (The plural is **vertices**).
- A line is called an **edge**.
- The number of edges that are attached to a vertex is called the **degree** of the vertex.

Exercise 5. The vertices of the graph below are labelled with the letters A, B, C, D . Write the degree of each vertex.



A:
 B:
 C:
 D:

Exercise 6. Complete the table below:

| | | | | | | | | |
|-------------------------------------|--|--|--|--|--|--|--|--|
| Graph Number | | | | | | | | |
| Euler Path? | | | | | | | | |
| Number of Vertices | | | | | | | | |
| Number of Vertices with even Degree | | | | | | | | |
| Number of Vertices with odd Degree | | | | | | | | |

Exercise 7. Complete the following sentence: “A graph has an Euler path if and only if ...”

Exercise 8. Can you explain why your answer to Exercise 7 is correct? Write your reasoning in the space below.

REVIEW: Complete the following sentences. You may refer to the rest of this packet.

A **graph** is:

A **vertex** is:

A **edge** is:

The **degree** of a vertex is:

Exercise 9. Here's a problem you might have seen your friends post on Facebook, called the "Five Rooms Puzzle": Draw a path, starting anywhere, that goes through each opening exactly once. Can you do it? (*Hint: Draw a graph that corresponds to the puzzle.*)

